Beiträge

Bernd Jähne* and Martin Schwarzbauer

Noise equalisation and quasi loss-less image data compression – or how many bits needs an image sensor?

Rauschäquivalisierung und quasi verlustfreie Bilddatenkompression – oder wie viele Bit benötigt ein Bildsensor?

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Abstract: Modern high-end image sensors require up to 16 bit quantisation. Uniform quantisation is, however, not well adapted to the signal, because the temporal noise strongly increases with the grey value. Here a non-linear transform \( h(g) \) is proposed, which yields an image sensor signal with an adjustable, grey-value independent temporal noise. The number of bits required to represent the noise-equalised signal is in good approximation equal to the maximum signal-to-noise ratio (SNR\(_{\text{max}}\)). Thus the noise-equalised signal of any imaging sensor with a full-well capacity of less than \( 2^{16} \) can be quantised with 8 bit or less. The only disadvantage is an insignificant increase in the overall noise level caused by additional quantisation noise.

Keywords: Image sensor, noise, quantisation, standard EMVA 1288.

1 Introduction

Early image signals were digitised with only 8 bit resolution. This is no longer sufficient with modern image sensors. With state of the art scientific CMOS (sCMOS) image sensors \([2]\) the standard deviation of the noise is only about 1 electron, while the full-well capacity is in the order of 30 000 electrons or higher. In order to cover the wide dynamic range from 1 to more than 30 000 electrons 16 bit quantisation is required. It can be expected that in the near future sensors will be available with the same low dark noise but an even higher full-well capacity. For such sensors even a 16 bit quantisation would no longer be sufficient. This would extend image data to 32-bit going in hand with a doubling of storage space and the need of faster data interfaces caused by the higher bandwidth.

There is another significant disadvantage. Image sensors with such a low dark noise are dominated by photon noise. Therefore the variance of the noise is about equal to the mean number of accumulated charge units. Accordingly the standard deviation of the noise increases with the square root of the mean digital value. Any equidistant quantisation is a clear mismatch in this situation. The quantisation must be fine enough to resolve the small random fluctuations at low signal levels but therefore it is...
much too fine at high signal levels. As an illustration the standard deviation of the noise of the pco.edge 5.5 sCMOS camera from PCO AG is shown in Figure 1. It covers more than two orders of magnitude. It is about 4 for the dark image but more than 300 close to saturation. Thus the quantisation is more than two orders of magnitude too fine in the bright parts of the image and more than 8 bits just carry the photon noise of the data.

Thus the question arises, whether it is possible to present the image signal in a different way. It would be ideal if the temporal noise level is independent of the grey value. In this paper it will be shown that this can be achieved by a suitable non-linear grey value transform. With this approach also the minimum number of bits can be derived, which presents the noisy image signal without any significant loss of information. It would be especially useful, if an 8 bit quantisation could be achieved for 12 to 16 bit image data. Transforming a 16 bit image to an 8 bit image constitutes already a compression factor of 2.

The paper is organized in the following way. In Section 2 the necessary theoretical background about quantisation of noisy signals is addressed. In particular, the required ratio between the standard deviation of the noise to the quantisation levels is discussed. Then in Section 3, the non-linear transformation for noise equalisation is discussed and it is derived how many bits are required to represent a camera signal in a noise-equalised signal without significant signal degradation. Finally, Section 4 verifies the theoretical considerations by a practical implementation with the pco.edge sCMOS camera.

2 Quantisation of noisy signals

Quantisation limits the resolution of output values. As simple as it appears at first glance, it is not easy to describe the effects of quantisation theoretically. This is due to the fact that quantisation is a non-linear and non-invertible function

$$q = \text{floor} \left( \frac{g + 0.5}{\Delta g} \right),$$

mapping real numbers $g$ to integer numbers $q$. The equidistant quantisation intervals are denoted by $\Delta g$. Therefore quantisation results in a staircase function (Figure 2). Every value $g$ within the interval $[q \Delta g - 1/2, q \Delta g + 1/2)$ results in the quantised value $q$. Only one thing is easy to state, the maximum error caused by quantisation of a signal. It is half of the quantisation interval $\Delta g$.

It is impossible to reconstruct the original values from the quantised values. This is in contrast to sampling. If a continuous signal is sampled and the conditions of the sampling theorem [5, 8] are met (in simple terms, every periodic component contained in the signal is sampled at least twice per period), then it is possible to reconstruct the continuous signal exactly from the sampled signal.

Because of the nature of the quantisation function in Equation (1), there can be no theorem for quantisation equivalent to the sampling theorem. It is also not necessary, because any real signal is uncertain in any way by its random nature. The question is rather how fine the quantisation must be so that the statistical properties of the signal are not disturbed. A proper theory of quantisation must answer the question: can the probability density function (PDF) of the continuous signal be reconstructed from the PDF of the quantised signal? If this is the case statistical quantities such as the mean and the variance of the random signal can be estimated from the quantised signal without any error.

This problem has been studied in detail by Widrow and Kollar [9] and resulted in quantisation theorems that are very similar to the sampling theorem if we replace the continuous signal $g$ by its PDF. There are actually several
quantisation theorems [9, Section 4.3]. The first states the condition that the continuous PDF can exactly be reconstructed from the PDF of the quantised signal, the second states the condition that only the moments of the continuous PDF can be exactly reconstructed from the PDF of the quantised signal. The latter conditions are less stringent and are of more importance, because the PDF normally is known and all what needs to determined is the mean and the variance of the signal. The conditions for exact reconstruction require that the PDF is band-limited, for details see Widrow and Kollar [9, Section 4.3]. If these conditions are met the mean of the quantised signal is exactly the mean of the original signal. The quantisation adds pseudo noise (PQN model, [9, Section 4.2]) to the quantised signal with a uniform distribution and the variance

$$\sigma_q^2 = \frac{1}{12}(\Delta g)^2.$$  \hspace{1cm} (2)

The variance of the original signal, $\sigma_g^2$, is then given from the measured variance of the quantised (sampled) signal, $\sigma_s^2$, by

$$\sigma_g^2 = \sigma_s^2 - \frac{1}{12}(\Delta g)^2.$$  \hspace{1cm} (3)

The big difference to sampling is that this condition is, unfortunately, never met exactly, neither by the normal distribution nor by the Poisson distribution, because both distributions are not strictly band-limited. Therefore Equation (3) will only be an approximation. The question then is just, how fine the quantisation must be so that the error is negligible. This is discussed in detail by Widrow and Kollar [9, Chapter 5]. The errors both in the mean and the variance computed by Equation (3) are negligible when

$$\Delta g \leq 2\sigma_g \quad \text{or} \quad \sigma_g \geq \Delta g / 2.$$  \hspace{1cm} (4)
This means that the standard deviation of the noise \( \sigma_g \) can be as low as half of the quantisation resolution \( \Delta g \).

These theoretical results were verified by independent numerical Monte Carlo simulations. For the simulations 201 mean grey values equally distributed between 0 and 1 were taken and zero-mean normally distributed noise was added to the values. The estimated mean and variances were averaged over 900 000 realizations of each value. Finally, the deviations in the estimations were averaged over all 201 values. The results are shown in Figure 3 for a range of \( \sigma_g / \Delta g = [0.3, 1] \). The mean grey value can be estimated with a maximum error of less than 0.014 DN even for standard deviations \( \sigma_g \) as low as 0.4 DN. The dimensionless units DN denotes a digital number. The maximum error of the estimate of the standard deviation remains below 0.04 even for standard deviations as low as 0.4.

3 Noise variance equalisation

3.1 General solution for noise variance equalisation

As already stated in Section 1, the variance \( \sigma^2_g(g) \) is a function of the mean grey value. First it is shown that for any monotonous function \( \sigma^2_g(g) \) a non-linear grey value transform \( h(g) \) can be found so that the standard deviation is constant in the transformed signal \( h \). This procedure has first been proposed by [3]. It is derived in the following.

By the laws of error propagation (see, e.g., [7] or [5, Section 3.2]), the variance of \( h(g) \) is given in first order by

\[
\sigma_h^2 \approx \left( \frac{dh}{dg} \right)^2 \sigma_g^2(g). \tag{5}
\]

If \( \sigma_h^2 \) is set to be constant, Equation (5) can be rearranged to

\[
\frac{dh}{dg} = \frac{\sigma_h}{\sqrt{\sigma^2(g)}}. \tag{6}
\]

Integration yields

\[
h(g) = \sigma_h \int_0^g \frac{dg'}{\sqrt{\sigma^2(g')}}. \tag{12}
\]

The integration constant is chosen in such a way that \( h(0) = 0 \). Equation (6) clearly says that an analytical solution exists for any function \( \sigma^2_g(g) \) for which the integral can be expressed by an analytic function. Of course, it is also possible to solve Equation (6) numerically.

3.2 Specific solution for linear camera model

Now the specific solution for the linear camera model can be derived. Then the variance increases linearly with the mean grey value [1]:

\[
\sigma^2_g(g) = \sigma^2_0 + Kg, \tag{7}
\]

known as the photo transfer relation [4], where \( K \) is the camera system gain with units DN/e\(^{-} \) and \( \sigma^2_0 \) the variance of the dark signal in units DN.

The following equations become simpler with the following new variables

\[
\tilde{g} = \frac{g}{g_{\text{max}}} \quad \text{and} \quad \tilde{h} = \frac{h}{h_{\text{max}}}. \tag{8}
\]

as the fraction of saturation of grey values in the range \([0, g_{\text{max}}]\) and \([0, h_{\text{max}}]\), respectively, and the maximum variance as

\[
\sigma^2_{\text{max}} = \sigma^2_0(g_{\text{max}}). \tag{9}
\]

Then Equation (7) can be written as

\[
\sigma^2_g(\tilde{g}) = \sigma^2_0 + (\sigma^2_{\text{max}} - \sigma^2_0) \tilde{g}. \tag{10}
\]

With the linear variance function Equation (10), the integral in Equation (6) yields

\[
h(g) = \frac{2\sigma_h}{K} \left( \sqrt{\sigma^2_0 + Kg} - \sigma_0 \right). \tag{11}
\]

Of course, the function \( h(g) \) takes only integer values (Figure 4). This leads to the equation

\[
h(g) = \text{floor} \left[ \frac{2\sigma_h}{K} \left( \sqrt{\sigma^2_0 + Kg} - \sigma_0 \right) + 0.5 \right], \tag{12}
\]

but for the sake of simplicity, the conversion to the nearest integer will be left out in the following equations.

The free parameter \( \sigma_h \) is the grey value independent standard deviation of the transformed signal \( h \). It can be used to map the values of \( h \) into the interval \([0, h_{\text{max}}]\). The condition \( h(g_{\text{max}}) = h_{\text{max}} \) yields

\[
\tilde{h} = \frac{\sqrt{\sigma^2_0 + (\sigma^2_{\text{max}} - \sigma^2_0) \tilde{g}} - \sigma_0}{\sigma_{\text{max}} - \sigma_0}, \quad \sigma_h = \frac{h_{\text{max}}}{2} \cdot \frac{\sigma_{\text{max}} + \sigma_0}{g_{\text{max}}}. \tag{13}
\]

From this equation it can be derived how many bits are required to quantise the equalised signal \( h \). This is given by the value of \( h_{\text{max}} \), which can be expressed by

\[
h_{\text{max}} = 2\sigma_h \cdot \frac{g_{\text{max}}}{\sigma_{\text{max}} + \sigma_0} = 2\sigma_h \cdot \text{SNR}_{\text{max}} \cdot \frac{\sigma_{\text{max}}}{\sigma_{\text{max}} + \sigma_0}, \tag{14}
\]
where the maximum signal-to-noise ratio is defined as

$$\text{SNR}_{\text{max}} = \frac{g_{\text{max}}}{\sigma_{\text{max}}}. \quad (15)$$

Because the dark noise $\sigma_0$ is much smaller than the maximum noise $\sigma_{\text{max}}$, the correction by $\sigma_0$ is not significant. Moreover, this makes the number of required bits, $h_{\text{max}}$, only slightly smaller. This means that the number of required bits depends in first order only on the maximum SNR.

Using the threshold for the standard deviation of the noise, $\sigma_h = 0.5$, derived at the end of Section 2, yields

$$h_{\text{max}} \leq \text{SNR}_{\text{max}}. \quad (16)$$

For any camera with a full-well capacity of less than $2^{16} = 65536$ electrons, the SNR$_{\text{max}}$ is smaller than $2^8$ and thus only 8 bits are required to quantise the noise-equalised signal without any significant loss of signal quality.

Replacing $g_{\text{max}}$ in Equation (14) using Equation (7), $h_{\text{max}}$ can also be related to the system gain $K$:

$$h_{\text{max}} = 2\sigma_h \cdot \frac{\sigma_{\text{max}} - \sigma_0}{K} \approx 2\sigma_h \cdot \sqrt{\frac{g_{\text{max}}}{K}}. \quad (17)$$

### 3.3 Inverse relation and interval widths

In order to convert the noise-equalised signal $h$ back to the original signal $g$, it is required to know the inverse relation to Equation (13). This is given by

$$\hat{g} = \hat{h} \cdot \frac{\hat{h}(\sigma_{\text{max}} - \sigma_0) + 2\sigma_0}{\sigma_{\text{max}} + \sigma_0}. \quad (18)$$

Another important question is the width of the intervals in the original signal, $\Delta g$ in relation to the constant unit interval width of $h$. In order to compute the non-linear transform in integer arithmetic by a lookup table with sufficient
accuracy it is required that always several values of the
original signal $g$ fall into one bin of the transformed sig-
nal $h$. This relation can be derived by differentiating Equa-
tion (18) and using Equation (13), resulting in

$$
\Delta g = \frac{\sigma_0 + \bar{h} (\sigma_{\text{max}} - \sigma_0)}{\sigma_h} = \sqrt{\frac{\sigma_0^2 + (\sigma_{\text{max}}^2 - \sigma_0^2) \bar{g}}{\sigma_h}},
$$

(19)

This means that $\Delta g > 1$ provided $\sigma_0 > \sigma_h$. The width $\Delta g$ of the intervals can also be seen for the example of the 16 bit pco.edge camera in Figure 4.

### 4 Practical application and tests

The non-linear grey value transform scheme was applied
to the scientific CMOS camera pco.edge 5.5 operated in fast-
scan mode. These cameras were used as a stereo reference
set-up (Figure 5) to acquire ground truth for driver assis-
tance systems [6].

The original linear output signal of the pco.edge is
quantised with 16 bit ($g_{\text{max}} = 2^{16}$). Using the procedures
according to the EMVA 1288 standard [1], the standard de-
viation of the dark noise and the system gain were mea-
sured (upper row in Table 1). With these values $\sigma_0 = 0.67$
was used to map the 16-bit space onto an interval $[0, 244]$. Therefore only 8 bits are required to represent the noise-
equalised signal.

Figure 4 directly shows the interval widths of $g$ that
are mapped to one value in the noise-equalised signal $h$.
It can be seen that between 6 to more than 700 values are
mapped to one $h$ value. Therefore no significant rounding
errors should occur.

Because it appears hard to believe that the signal of
a 16-bit camera can be represented by just 8 bits, a critical
test was performed. A pco.edge camera (SN 1013) was mea-
sured according to the EMVA 1288 standard. The EMVA
1288 characterize the sensitivity, linearity, and temporal
noise of the camera. If the compression to a non-linear
8 bit signal would cause a degradation it should show up
in the EMVA 1288 measurements.

The results of the EMVA 1288 measurements with the
original 16 bit signal are contained in the first row of Ta-
ble 1. These values where taken to compute a compression
lookup table (LUT) with $2^{16}$ entries and 8 bit values and $\sigma_h = 0.67$.

After applying the non-linear compression LUT, almost constant, signal independent noise is achieved (Fig-
ure 6) as predicted by the theoretical considerations. The
small deviations are caused by slight deviations from a lin-
ear photon transfer curve. The measured standard devi-
ation is slightly higher than 0.67, because of the addi-
tional quantisation noise with a variance of $1/12$. Addi-
tion of the variances yields a resulting standard deviation
of 0.73 DN, which is 8.9% higher and in good agreement
with the measured values, shown in Figure 6.

The influence of the compressing LUT to the EMVA
1288 parameters was tested in the following way. Firstly,
the non-linear compression LUT is applied and then the in-
verse LUT. These two processing steps result again in 16 bit
values, but now at most 256 values are occupied in the
16-bit space. With images processed in this way, a second
EMVA 1288 measurements is performed.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Measurement No.</th>
<th>$\eta$</th>
<th>$\sigma_0$ (e$^-$)</th>
<th>$\sigma_h$ (e$^-$)</th>
<th>$K$ (DN/e$^-$)</th>
<th>$g_0$ (DN)</th>
<th>$\text{SNR}_{\text{max}}$</th>
<th>DR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct</td>
<td>m0387</td>
<td>0.454</td>
<td>3.91</td>
<td>1.97</td>
<td>1.975</td>
<td>96.32</td>
<td>185</td>
<td>12570</td>
</tr>
<tr>
<td>$\sigma_h = 0.67$</td>
<td>m0389</td>
<td>0.385</td>
<td>4.36</td>
<td>1.86</td>
<td>2.334</td>
<td>96.15</td>
<td>172</td>
<td>11530</td>
</tr>
<tr>
<td>Corrected</td>
<td></td>
<td>0.455</td>
<td>4.36</td>
<td>2.21</td>
<td>1.975</td>
<td>96.15</td>
<td>172</td>
<td>11530</td>
</tr>
</tbody>
</table>

Figure 5: Reference stereo set-up with two pco.edge 5.5 cameras to acquire ground truth for driver assistance systems with 960 x 2560 pixel at 200 fps.

Table 1: EMVA 1288 measurement results for the pco.edge. Upper row: original linear 16-bit results; middle row: results from 16 bit signal after compression to and decompression from non-linear 8 bit signal; lower row: after correction for system gain $K$ according to additional quantisation noise. Symbol list: quantum efficiency $\eta$, standard deviation of dark signal $\sigma_0$, system gain $K$, dark signal $g_0$, maximum signal-to-noise ratio $\text{SNR}_{\text{max}}$, dynamic range DR.
The results are shown in the second row of Table 1. The influence of the additional quantisation noise yields a higher gain value $K$. Because $K$ scales with the variance, it should be 18.6% higher. The measurements show that it is 18.1% higher, which is an excellent agreement. If the correct gain is used, see third row in Table 1, the same quantum efficiency is computed as with the unprocessed images (row one in Table 1).

The dark noise $\sigma_0$ remains 11.5% higher. Because of the additional quantisation it should be 8.9% higher. Likewise the maximum SNR and dynamic range DR should be 8.9% lower. The measured values are 7.6%, and 9.0% lower, respectively. Given the slight deviations from a linear camera response and a linear photon transfer curve, the agreement between measurements and prediction is very good.

### 5 Discussion, conclusions, and outlook

In this paper it is shown that an equidistant quantisation of a linear camera signal is not a good choice. It is much more suitable to apply a non-linear transform to obtain a grey-value independent standard deviation of temporal noise. In this way, much less bits are required to represent the image signal. The maximum signal-to-noise ratio determines how many bits are required, see Equation (17).

The only disadvantage of this approach is a slightly higher noise level due to additional quantisation noise. However, this effect is not large. According to Equation (3) the relative increase in the standard deviation of the temporal noise can be estimated to be

$$1 + \frac{1}{24\sigma_h^2}. \tag{20}$$

Even for the limiting case $\sigma_h = 0.5$, the increase in the standard deviation is only 17%, for $\sigma_h = 1$ it is just 4%.

The noise-equalised presentation of image signals has further significant advantages.

- Firstly, many image processing algorithms expect a constant, grey value independent standard deviation of the temporal noise. Because of the linear increase of the temporal noise variance with the grey value this is by far not the case for any image sensor. In other words, the signal has a much better SNR in the dark parts of the image than expected from a grey value independent noise level. For a sensor with negligible dark noise, the SNR at 1% saturation decreases only by a factor of 10, while for a grey value independent noise level it would decrease by a factor of 100. With the proposed noise-equalisation, computation of error propagation is much easier.

- Secondly, by transforming the signals of different cameras to a non-linear signal with a given constant standard deviation $\sigma_h$ of the temporal noise, all cameras show the same temporal noise independent of their parameters such as system gain, temporal dark noise and maximum SNR. In this way it would be possible to change cameras without the need to adapt the image processing algorithms to the camera parameters. Therefore the noise equalisation proposed in this paper has the potential for a normalized and univer-
sal camera signal. This approach is, of course, only valid if the spatial signal non-uniformity, i.e., the dark signal non-uniformity (DSNU) and the photoresponse non-uniformity (PRNU) is significantly smaller than the temporal noise. This needs further investigations.

Thirdly, the noise equalisation compresses the image signal. Without any further processing, a 16 bit signal is already compressed into a 8 bit signal, resulting in a compression factor of two. For the stereo reference system (Figure 5) used to acquire ground truth for driver assistance systems [6] this compression made it possible to store the signals of two pco.edge camera running at 200 fps with a resolution 960 × 2560 pixel in real-time to memory and to reduce the storage capacity by a factor of two.

The potential for further lossless compression is high. This is due to the fact that standard deviation of the temporal noise is about the same as the quantisation interval. Therefore temporal noise is confined to just a few bits independent of the grey value. This is not the case for the original linear signal. There the standard deviation of the temporal noise can be as high as 400. In order to get a quick estimate of the potential for further compression, 16 bit images with a constant image and $\sigma_g = 400$ and 8 bit images with a constant image and $\sigma_h = 0.7$ were compressed with the LZW and ZIP algorithms implemented in the open source TIFF library. For the 16 bit image with $\sigma_g = 400$ the compression factor was only up to 1.24, while for the 8 bit image with $\sigma_h = 0.7$ the compression factor was up to 3. In combination of noise equalisation and further lossless compression schemes it appears therefore likely that compression factors of more than 4 can be achieved.

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References


Bionotes

Bernd Jähne
Heidelberg University, Heidelberg
Collaboratory for Image Processing (HCI),
Speyerer Straße 6, 69115 Heidelberg,
Germany
Bernd.Jaehne@iwr.uni-heidelberg.de

Bernd Jähne received his Diploma, Doctoral degree and Habilitation degree in Physics from Heidelberg University in 1977, 1980, and 1985, respectively, and a Habilitation degree in Applied Computer Science from the University of Hamburg-Harburg in 1992. From 1988 to 2003 he hold a research professorship at the Scripps Institution of Oceanography, University of California in San Diego. Since 1994 he is professor at the Interdisciplinary Center for Scientific Computing (IWR) and Institute for Environmental Physics of Heidelberg University and since 2008 he also heads the Heidelberg Collaboratory for Image Processing (HCI). His research interests include small-scale air-sea interaction and image processing. He chairs the EMVA 1288 committee of the European Machine Vision Association for camera characterization.
Martin Schwarzbauer
PCO AG, Donaupark 11 93309 Kelheim, Germany
Martin.Schwarzbauer@pco.de

Martin Schwarzbauer received his Diploma degree in Electrical and Information Engineering and M.Eng. degree in Electrical and Microsystems Engineering from the University of Applied Sciences, Regensburg, Germany, in 2007 and 2010, respectively. He joined PCO AG, Kelheim, Germany, in 2007 as an R&D hardware engineer. His main focus at PCO started with data interfaces. Later he joined as a founding member the CameraLinkHS standardization committee hosted by the Automated Imaging Association (AIA, Ann Arbor, Michigan, USA) in 2010. Meanwhile he is developing complete camera designs for high-speed and scientific cameras. In 2014 he has started working towards the Ph.D. degree. His research interests are real-time image compression algorithm especially for very high dynamic image data and sensor RAW data.